## Hexakuro Instructions

Hexakuro ${ }^{\text {TM }}$ is a hexagonal implementation of the number puzzle Kakuro (or Cross Sums). The object of the puzzle is to fill in each blank hexagon with a digit from 1 to 9 so that no digit is repeated in a continuous line of vertical or diagonal white hexagons. In addition, the digits in the line of hexagons must add up to the clue which is shown in a triangle at one end of the line. Unlike in Kakuro, where there is a clue for each row and column (on a rectangular grid), in Hexakuro many clues may be missing. In some cases, a hexagon may have no associated clue at all and the solution relies on your Sudoku skills based on having no number repeat in the contiguous vertical and diagonal hexagons. A well designed puzzle should have only a single solution.

In this introduction we will solve a very small puzzle to see how things work. (This is puzzle Hexakuro 101 in the Online Game Ap if you want to try it yourself). The solution to a slightly more involved example puzzle can be found here.


In this example a coordinate system is marked on the outside of the puzzle to help identify the hexagons. There are diagonals $A, B, C, D, E$ slanting downwards from left to right. There are diagonals $a, b, c, d, e$ slanting downwards from right to left. A hexagon at the intersection of two diagonals is labelled with both coordinates. For instance the top hexagon in the middle is hexagon Aa.

To start solving a puzzle, it is useful to look for hexagons that have two clues, with only one number being consistent with both of the clues.

For example, Hexagon $A a$ is in diagonal " $A$ " which has a sum of 3 for two hexagons. The only two digits that add up to 3 are in the set $\{1,2\}$. (That is, $1+2=3$ and $2+1=3$ ). Hexagon Aa is also in diagonal "a" which has a sum of 4 for two hexagons. The only sets of digits adding up to 4 are either $\{1,3\}$ or $\{2,2\}$. But $2+2$ is not allowed because the digits would have to repeat in the same diagonal. Since the only overlap between the set $\{1,2\}$ and the set $\{1,3\}$ is the number 1 , we can deduce that 1 is the solution for hexagon Aa. Then the solution for hexagon Ab must be 2 , and for hexagon Ba must be 3 as shown in the next figure.


If we look at the bottom part of the puzzle, we can see a similar situation to the one just discussed for hexagon Ee. Diagonal "E" has a clue that two hexagons must add up to 17, and the only possibility is the set $\{8,9\}$. Diagonal "e" has a clue that two hexagons must add up to 16 , and the only possibility with nonrepeating numbers is the set $\{7,9\}$. The only overlap between these sets is the number 9 , which must be the solution for hexagon Ee. It follows that the solution for hexagon Ed is 8 and for hexagon De is 7 . This is shown in the next figure.

Now look at the column going from Ab to De . This column has four hexagons, so we need four non-repeating digits that add up to the clue of 17. Two of these hexagons are already solved with the numbers 2 and 7 , which add up to 9 . Therefore, the other two hexagons ( Bc and Cd ) must have digits which add up to $17-9=8$. The sets of two numbers adding to 8 are $\{1,7\},\{2,6\}$, $\{3,5\}$ and $\{4,4\}$.

We can eliminate $\{4,4\}$ because repeating digits are not allowed. We can eliminate $\{1,7\}$ because we have already used a 7 . We can eliminate $\{2,6\}$ because we have already used a 2. That leaves us with $\{3,5\}$. It is useful to mark these in as "guesses" for the two hexagons. The app has a pull-down menu, or on a laptop you can use the control key before the guess number to mark it in. The puzzle with the guesses marked in is shown in the next figure.


Since the 3 is already used in diagonal " B " (at hexagon Ba ), it follows that the solution for hexagon Bc must be 5 . Then the only remaining solution for hexagon Cd is the number 3 .

Now we can use the clue for diagonal "B" which has three numbers which must add up to 14 . Since two of these numbers are $3+5=8$, the remaining number must be $14-8=6$. So the solution for hexagon Bb is 6 , as shown in the next figure.


Our example Hexakuro puzzle now has four remaining hexagons to solve for, and two remaining unused clues. Consider diagonal " c " which has three numbers adding up to 11. We have already solved for the 5 in hexagon Bc , so the other two hexagons must add to $11-5=6$. The sets of two numbers adding to 6 are $\{1,5\},\{2,4\}$ and $\{3,3\}$.

We can eliminate $\{3,3\}$ because it has repeating numbers. We can eliminate $\{1,5\}$ because we have already used the 5 . That leaves us with $\{2,4\}$ which have been marked in as guesses in the next figure.


In solving a Hexakuro puzzle, it will sometimes happen that two adjacent hexagons have the same two possible guesses, as has happened here for hexagons Dc and Cc with the guesses $\{2,4\}$. That means that another hexagon which is in the same diagonal or column as those two cannot have one of those two numbers as its solution. Specifically, the solution for hexagon Dd cannot be a 2 and cannot be a 4.

Now we can use our Sudoku skills to solve for hexagon Dd.
We can eliminate 1 because that is used in column Aa-Ee. We can eliminate 2 because that is in either Dc or Cc. We can eliminate 3 because that is used in diagonal " d ". We can eliminate 4 because that is used in either Dc or Cc. We can eliminate 6 because that is used in column Aa-Ee. We can eliminate 7 because that is used in diagonal " D ". We can eliminate 8 because that is used in diagonal " d ". We can eliminate 9 because that is used in column Aa-Ee. That leaves us with 5 as the solution for hexagon Dd.


Looking now at our last clue (diagonal "C"), we need three numbers which add up to 16 , one of which is a 3 , and the other of which is in the set $\{2,4\}$. The only possibilities are 16 -$3-2=11$ or $16-3-4=9$. Of course, 11 is not a single digit number, so the solution to hexagon Cb must be 9 and for hexagon Cc must be 4 .

Finally that leaves 2 as the solution for hexagon Dc, and we are done!


